An Open-Source RISC-V Vector Math Library

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An Open-Source RISC-V VecLibm

- Background and Motivation
- Snapshots of RISC-V Vector ISA
- RISC-V Vector Math Library: Strategies and Illustrations
- Rivos FP64 Vector Libm current status – at a glance

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- Two distinguished features: modular and extensible

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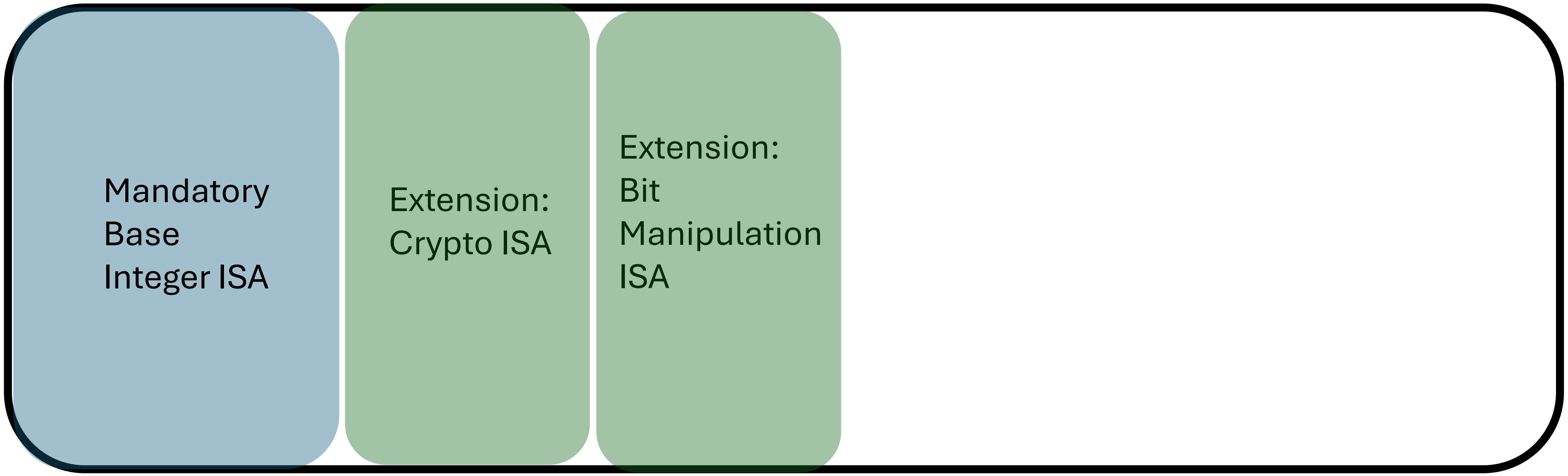

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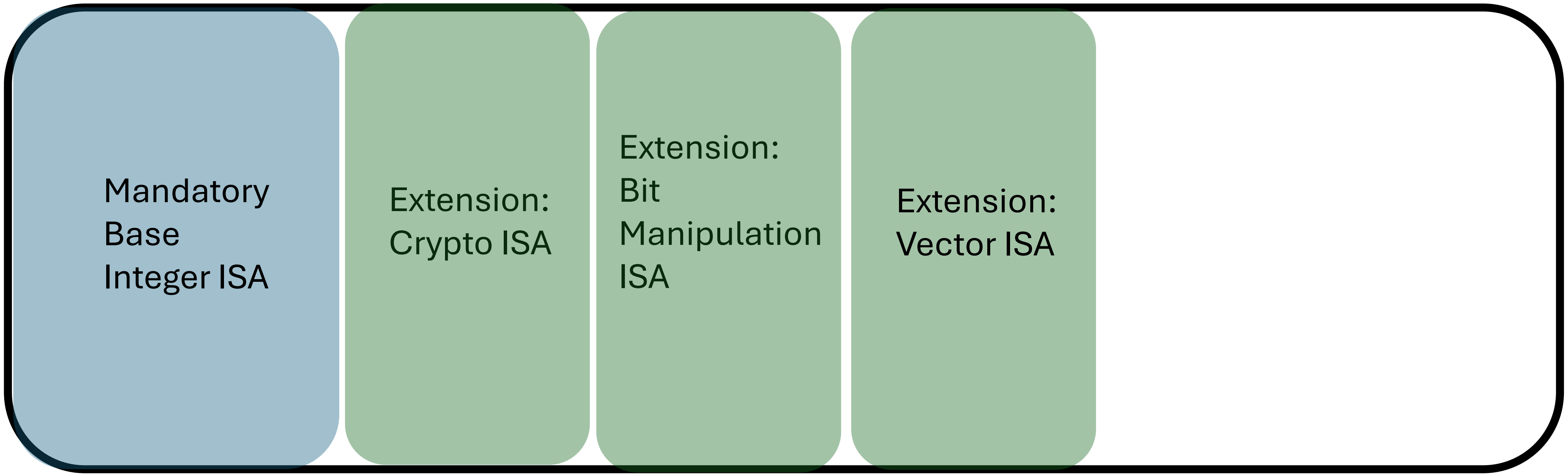

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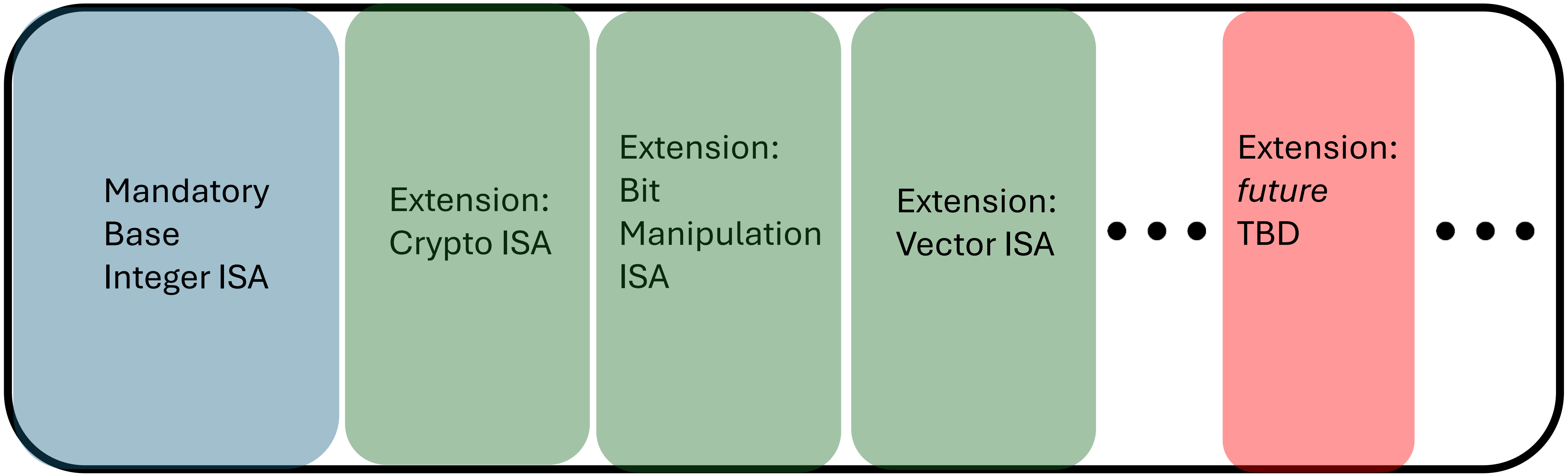

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In `math.h` `exp(x)`, `sin(x)`, `atan(x)`, etc.

Scalar:
```
double exp(double x);
```

VS.

Vector:
```
void vec_exp(int N, const double* x, double* y);
```

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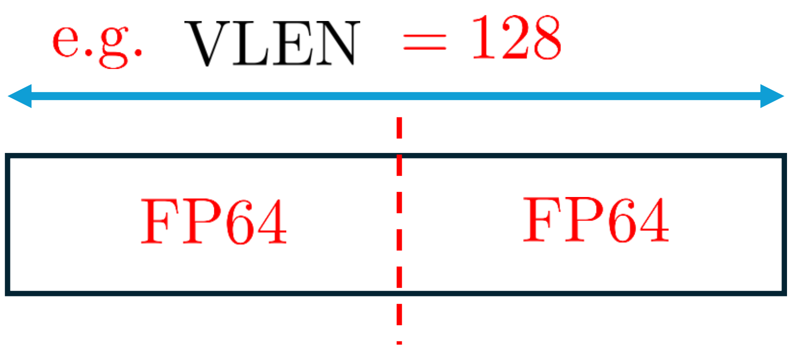
- Open RISC-V Vector Libms are worthy additions
  - FP64 vector libm fits the need to traditional computational science and HPC
  - Requires experience to construct a numerically reliable library
  - Scope is modest that a start up can undertake as a good-citizen project

Snapshots of RISC-V Vector ISA: *General*

32 architectural registers
VLEN implementation defined



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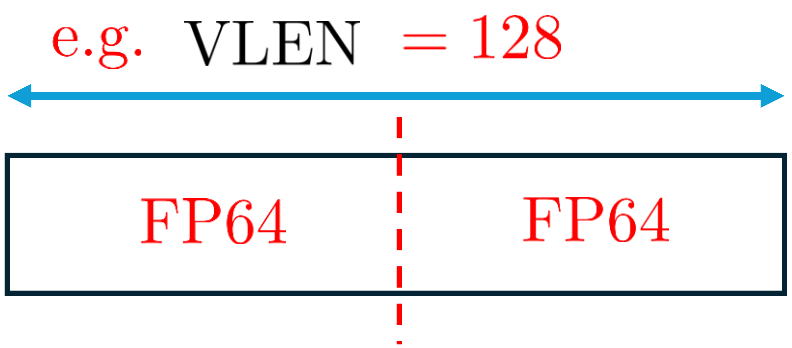

Registers are type agnostic:
Just a number of bits,
to be interpreted in the context
of the instructions

32 architectural registers
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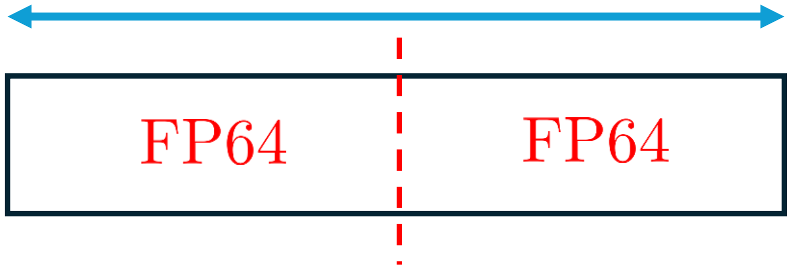
Can configure; group LMUL physical regs to form one logical vector reg.

e.g. VLEN $= 128$



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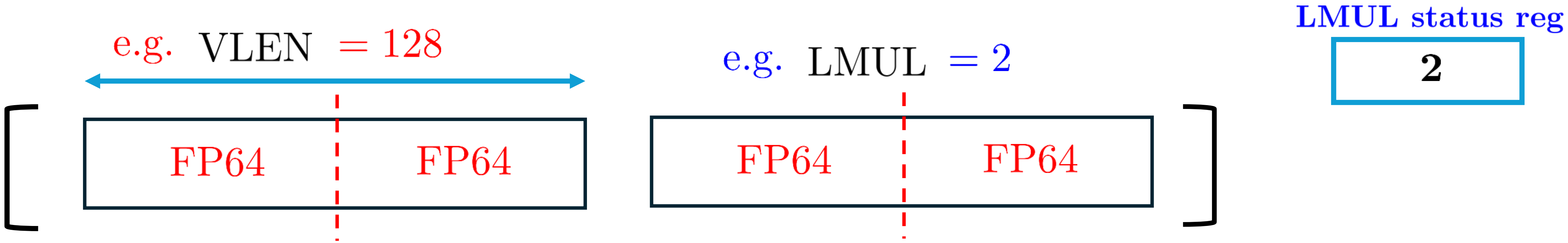

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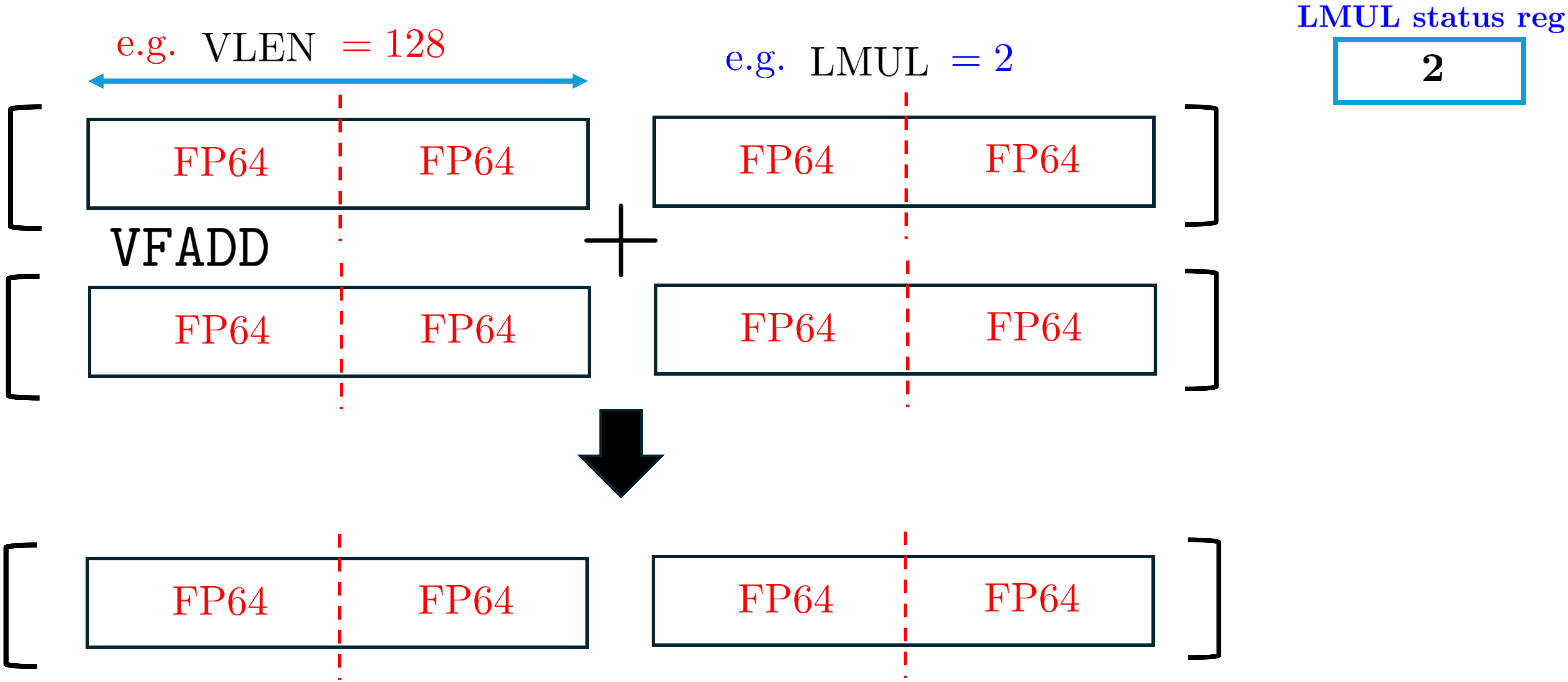

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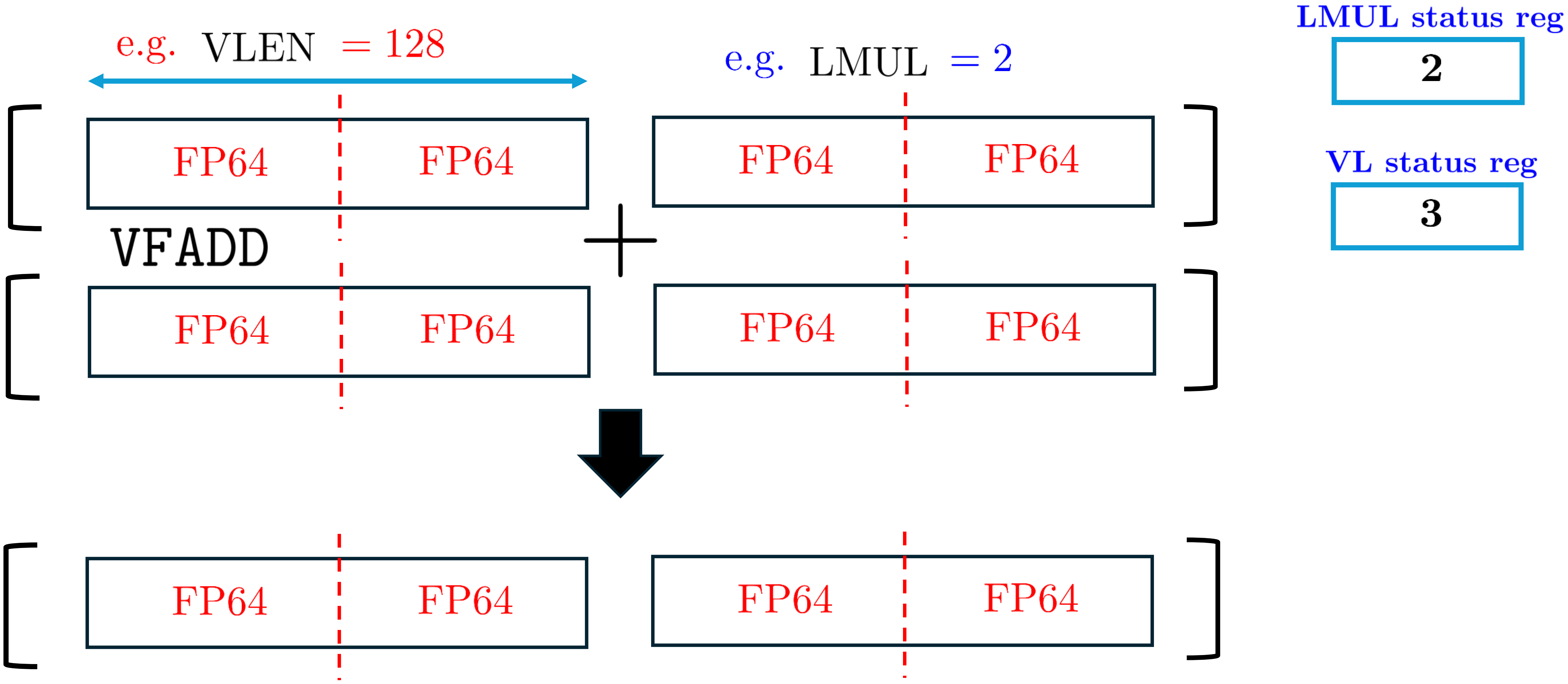

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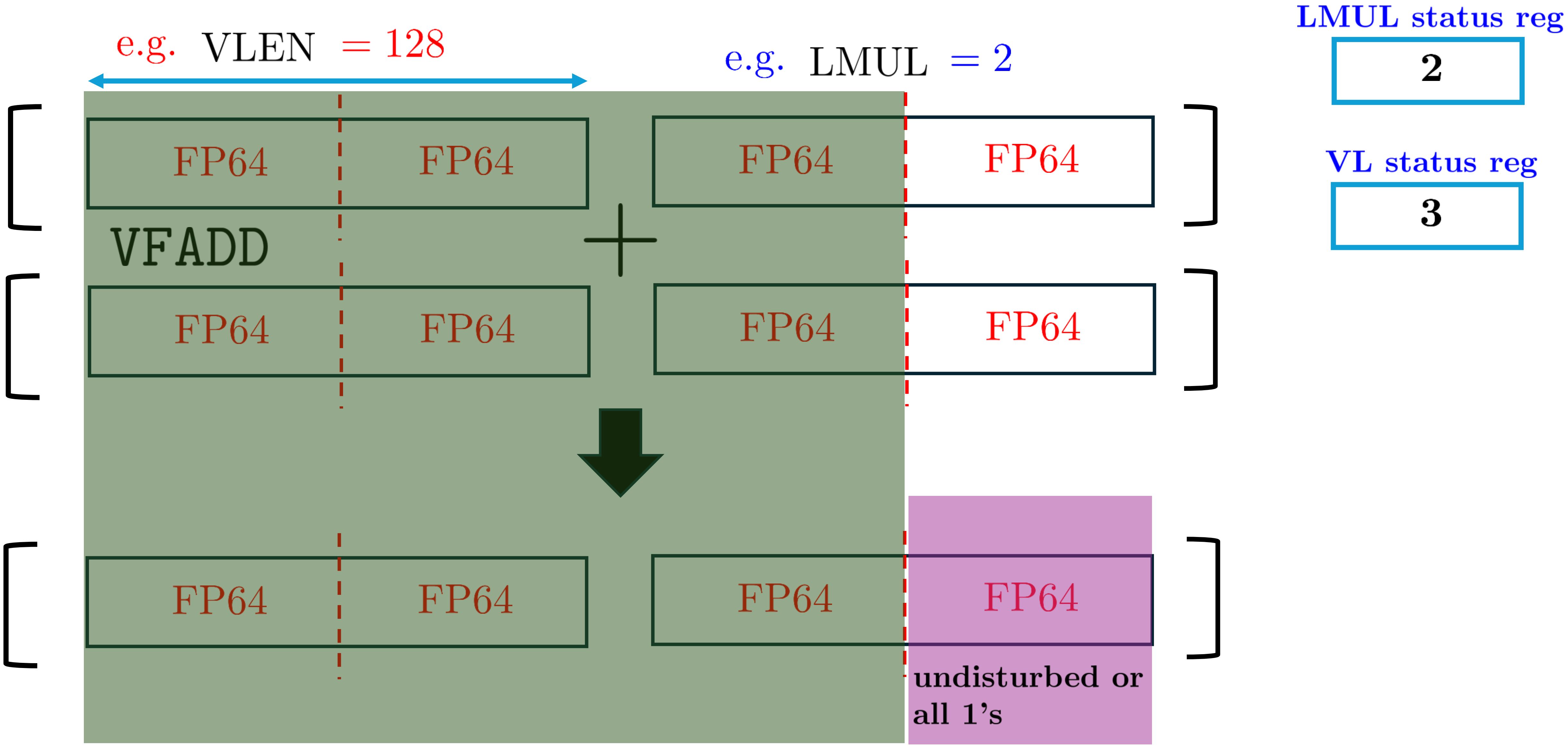

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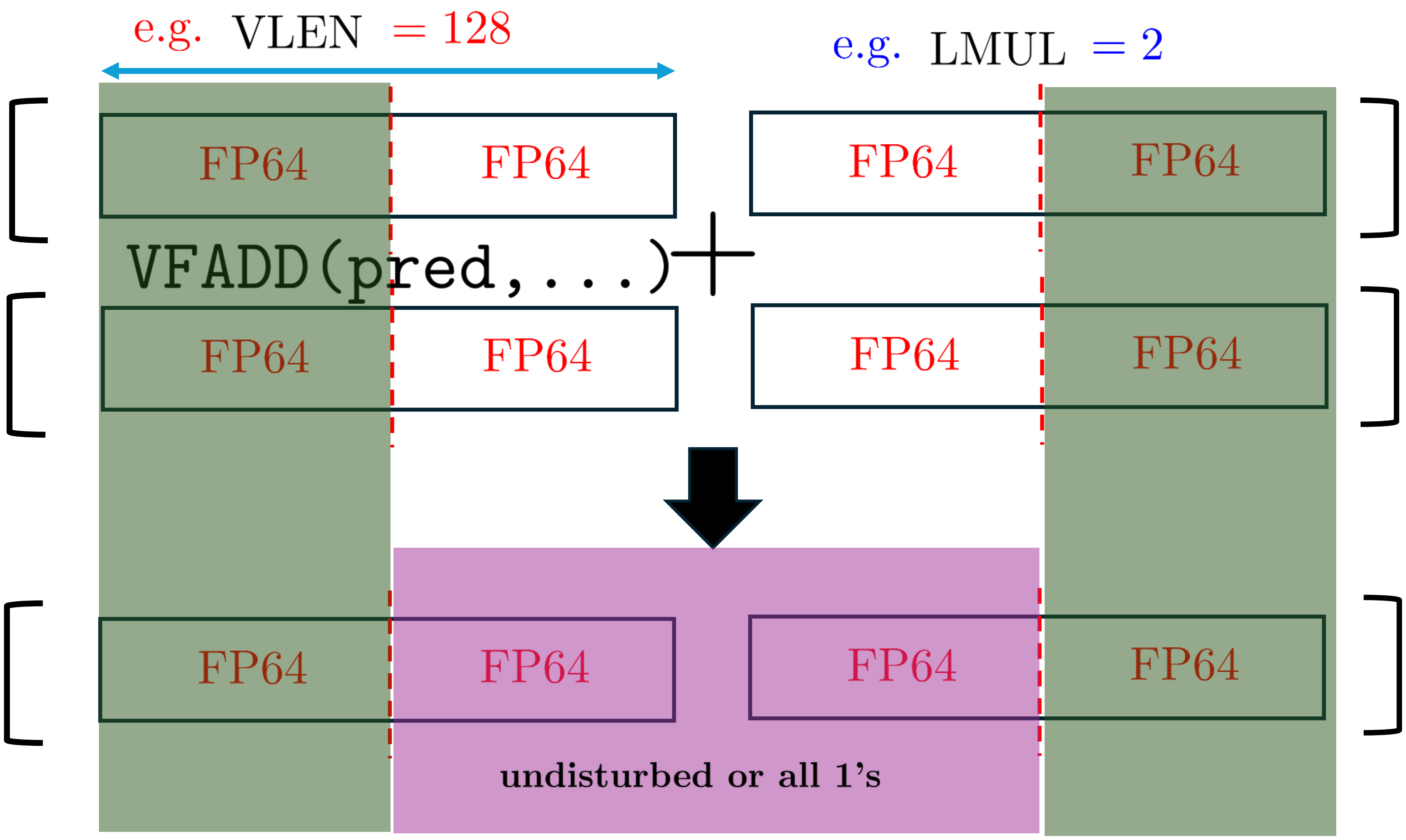


LMUL status reg

2

VL status reg

4

predicated execution

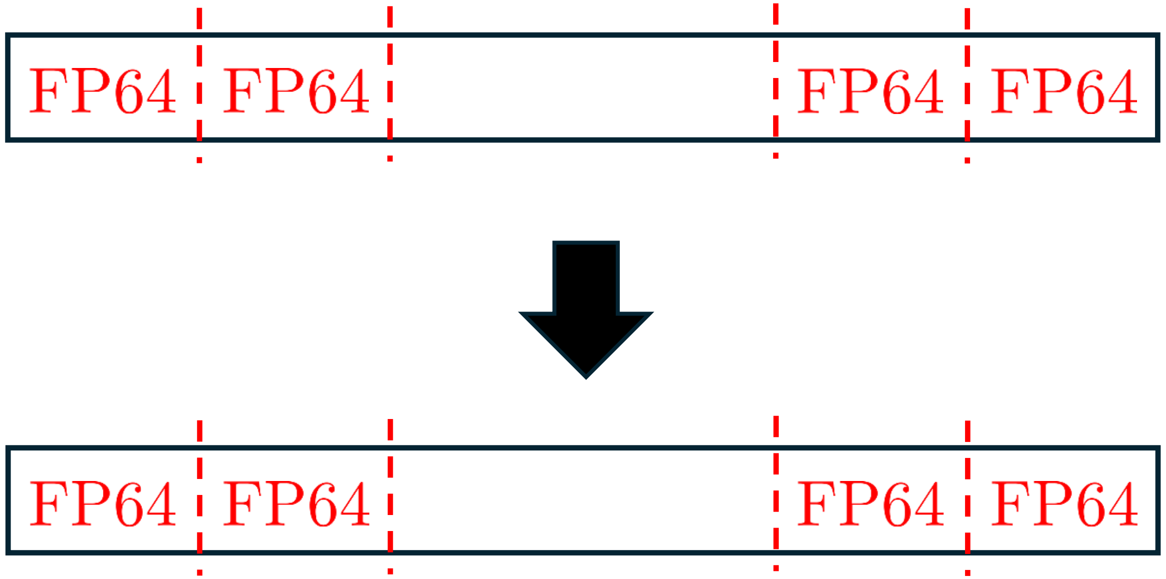
pred. reg

1 0 0 1 • • •

Snapshots of RISC-V Vector ISA: *Floating-Pt.*

Some Useful Vector Instructions for Veclibm

`VFREC7, VFRSQRT, VFSQRT, etc`



Unary FP instructions:
`vfrec7, vfrsqrt7, vfsqrt`

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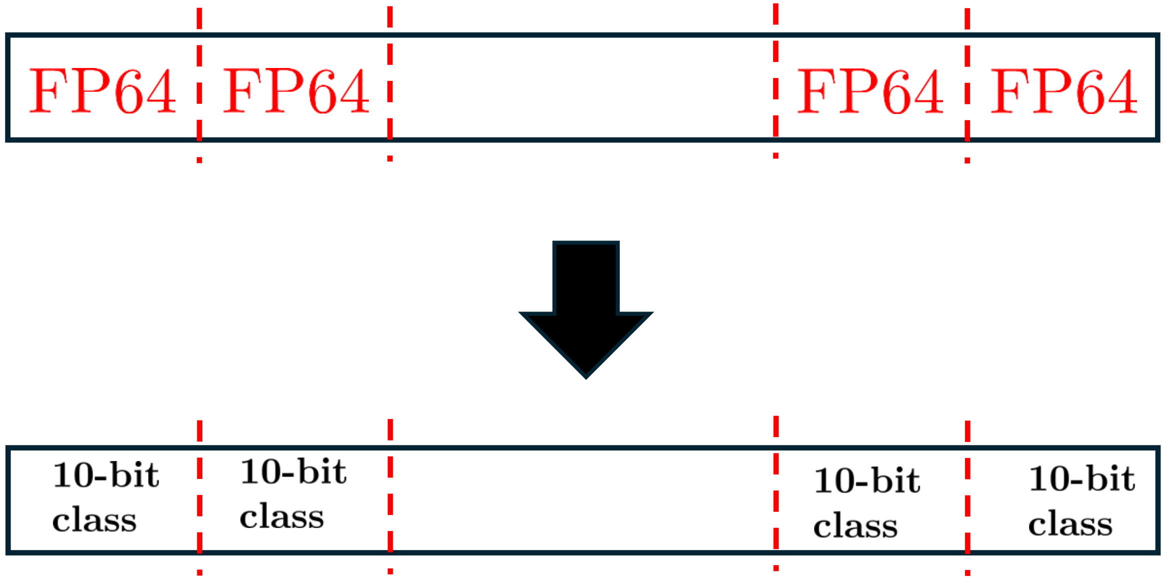
`VFCLASS`



Unary FP instructions:

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`vfclass`

rd bit	Meaning
0	*rs1* is $-\infty$.
1	*rs1* is a negative normal number.
2	*rs1* is a negative subnormal number.
3	*rs1* is -0.
4	*rs1* is $+0$.
5	*rs1* is a positive subnormal number.
6	*rs1* is a positive normal number.
7	*rs1* is $+\infty$.
8	*rs1* is a signaling NaN.
9	*rs1* is a quiet NaN.

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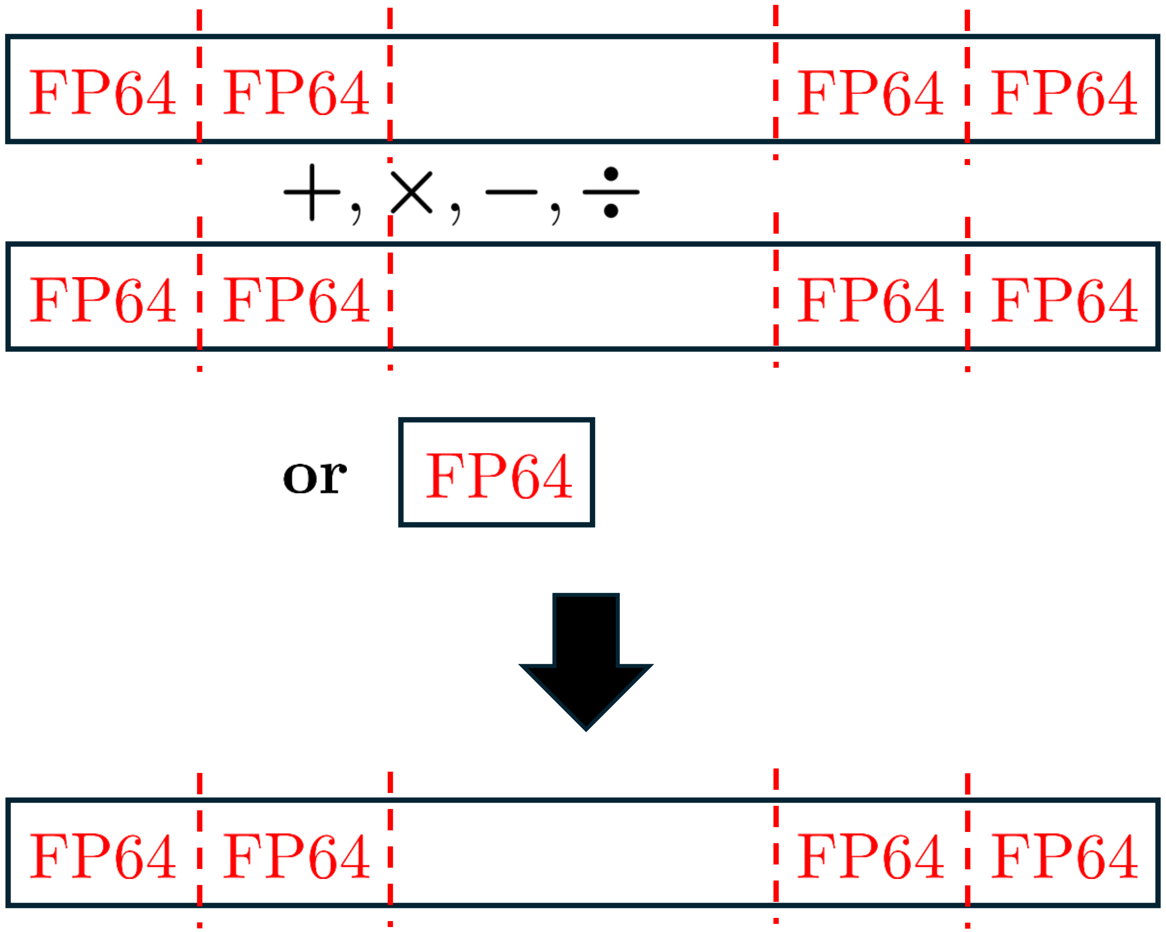

Unary FP instructions:

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Binary FP instructions:

`vfadd`, `vfmul`

`vfsub`, `vfdiv`

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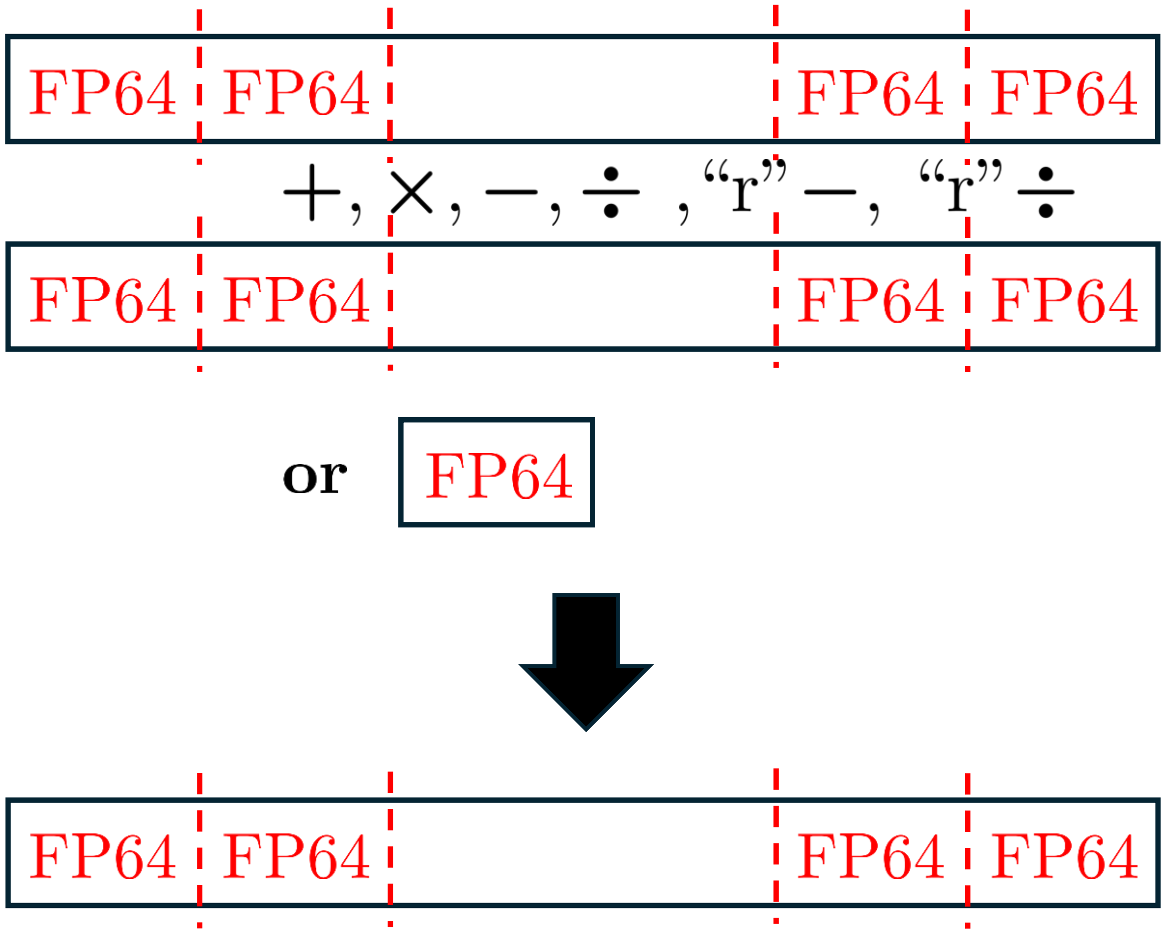

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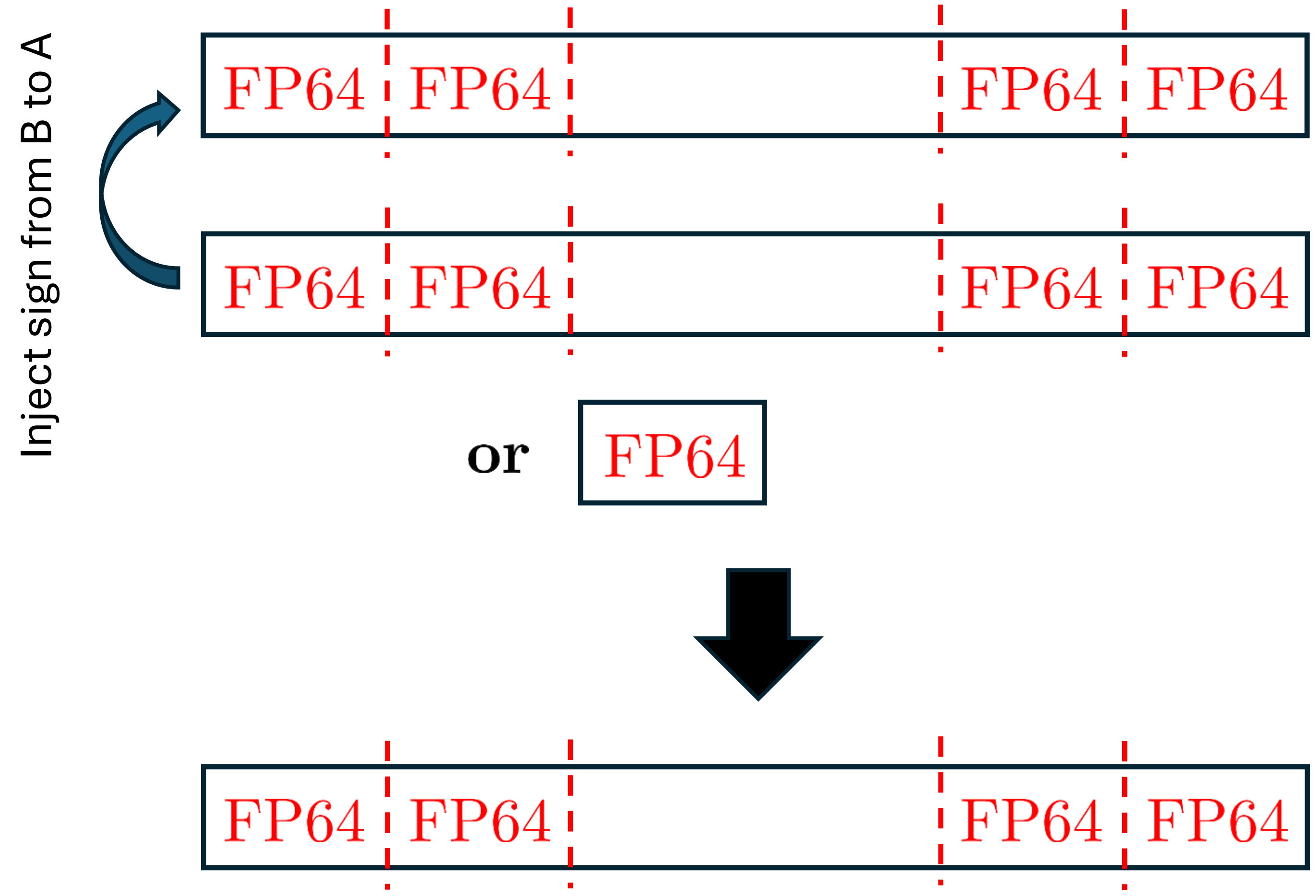

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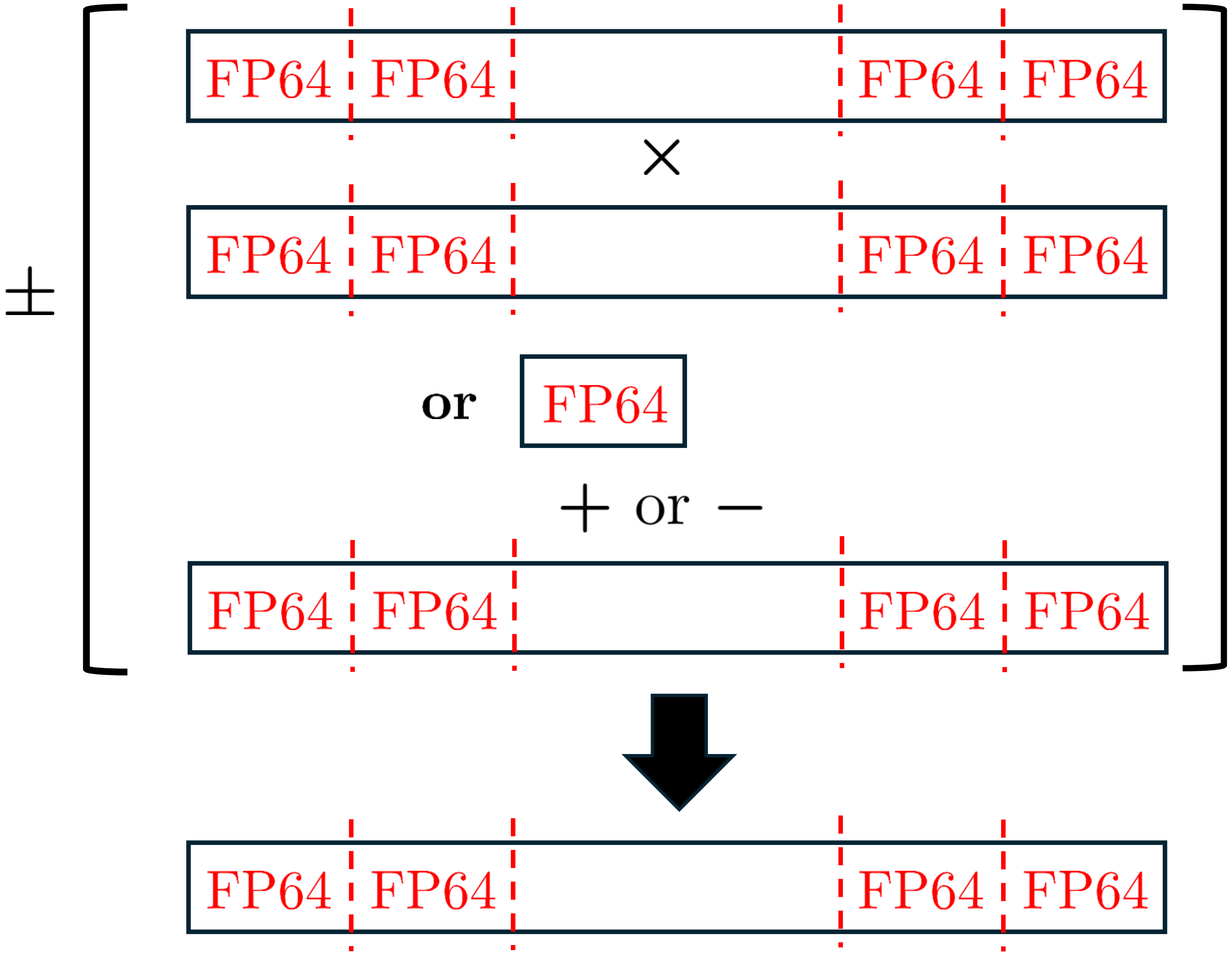

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`vfsub`, `vfdiv`
`vfrsub`, `vfrdiv`
`vfsgnj`, `vfsgnjn`, `vfsgnjx`

Ternary FP instructions:
`vf[n]madd`, `vf[n]msub`
`vf[n]macc`, `vf[n]msac`

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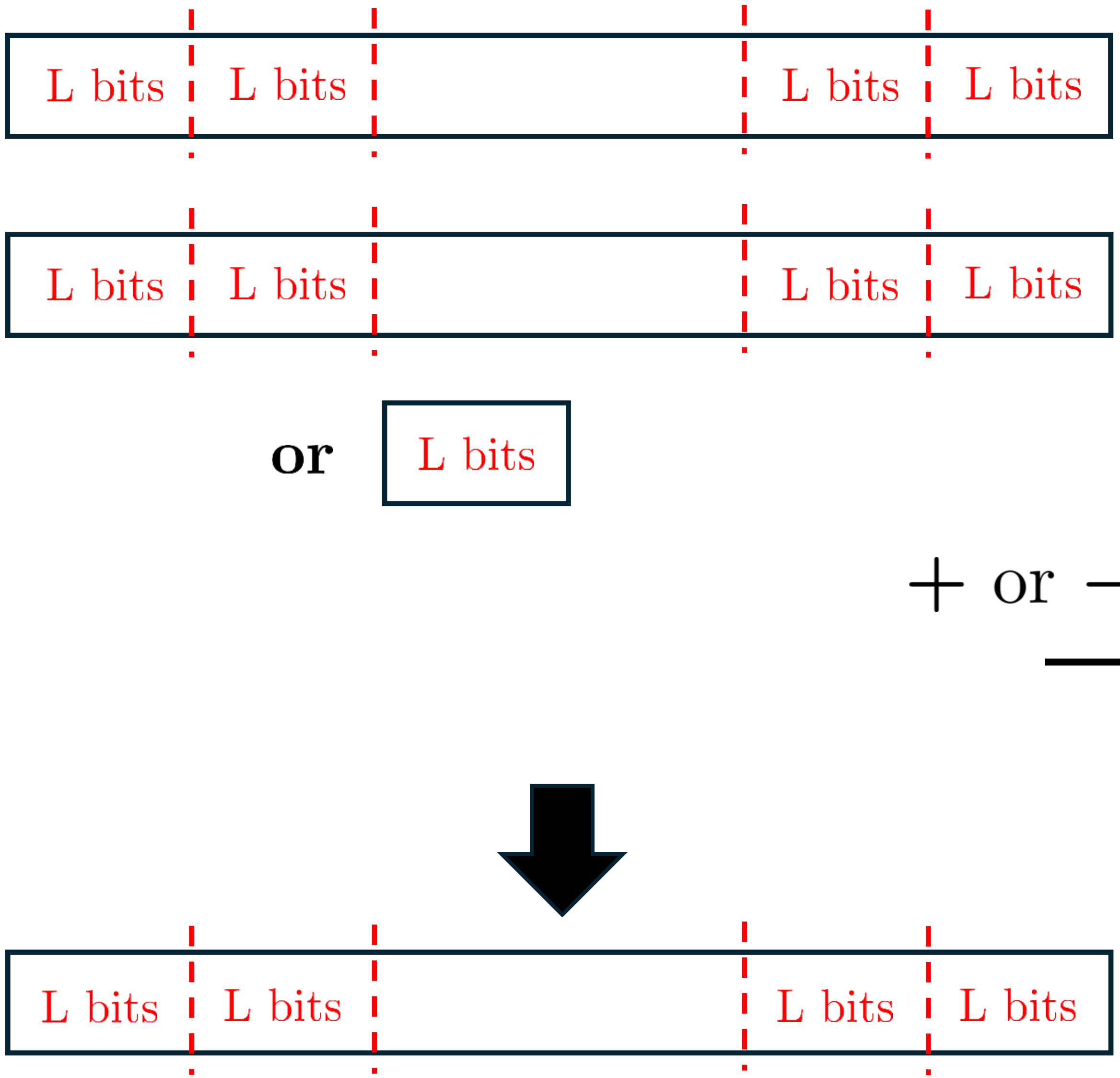
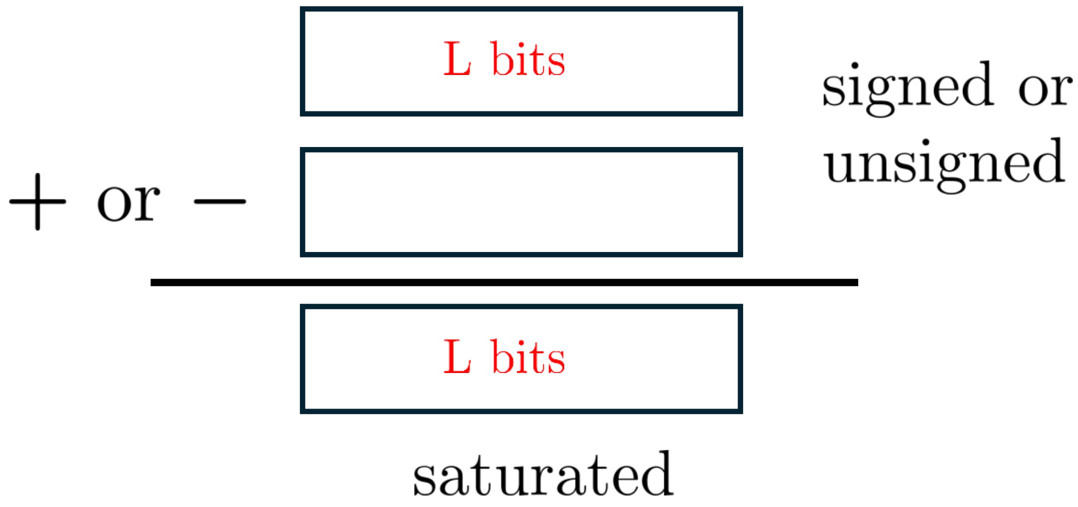


Fixed-point arithmetic:

`vsadd`, `vssub`, `vaadd`, `vasub`
`vsmul`



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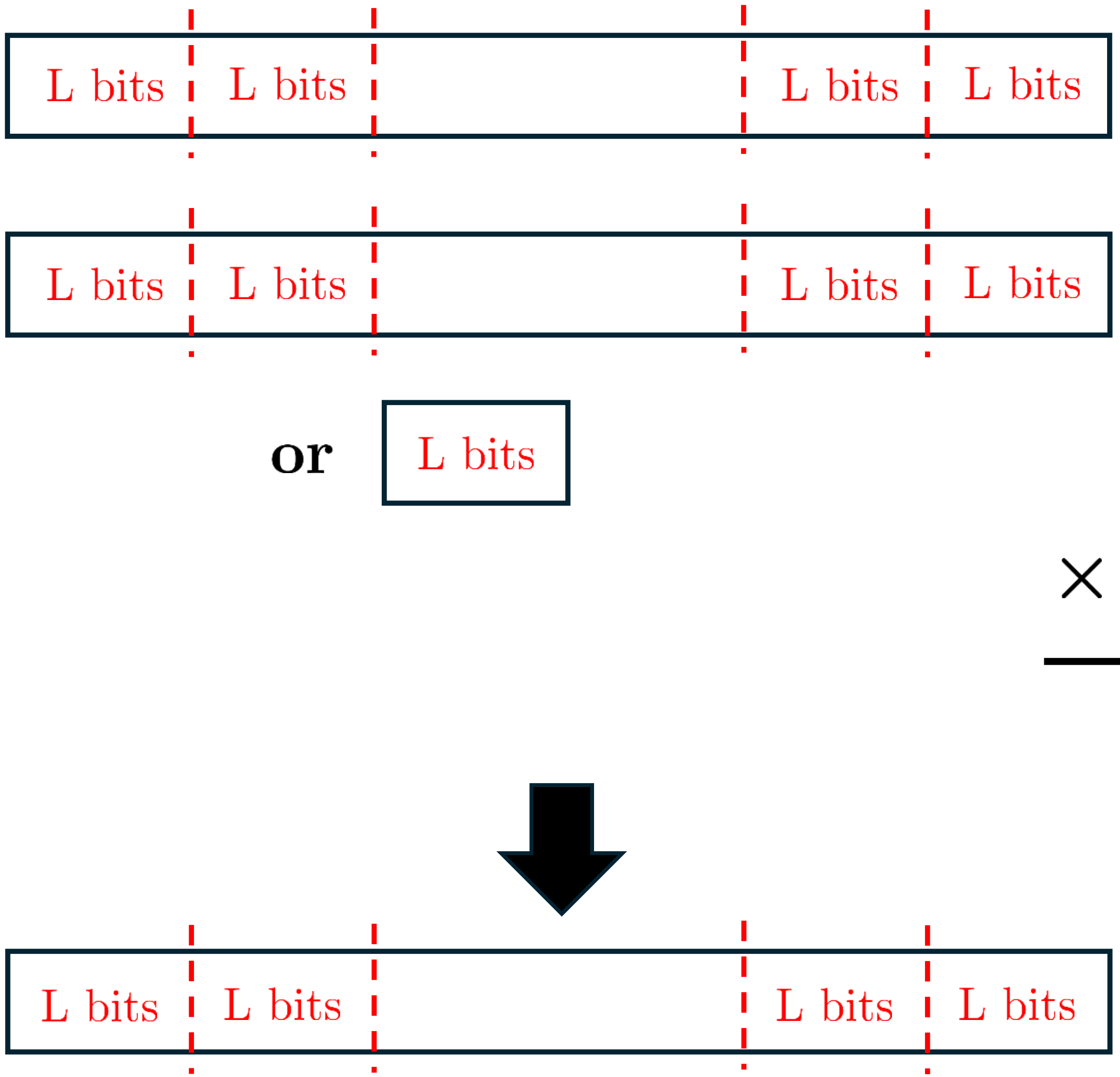

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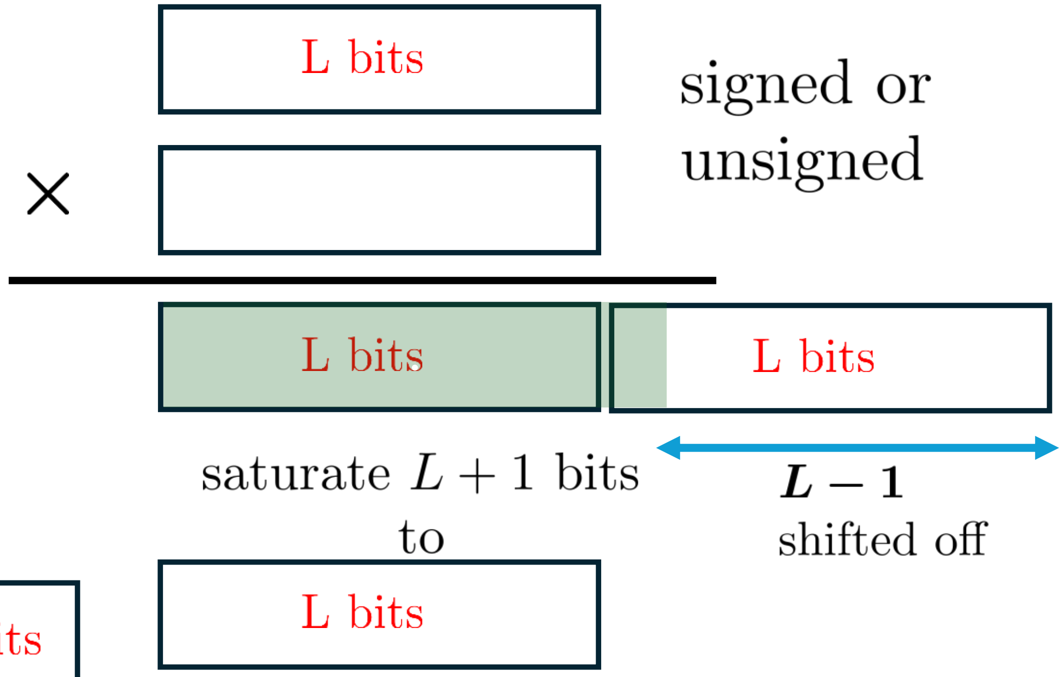

Some Useful Vector Instructions for Veclibm

$A = \text{int}(a \times 2^{\sigma_a})$ (A is a in Q-σ_a format)
$B = \text{int}(b \times 2^{\sigma_b})$ (B is b in Q-σ_b format)
$C := \texttt{vsmul}(A, B)$ C is ab in σ_c format
$\sigma_c = \sigma_a + \sigma_b - 63$

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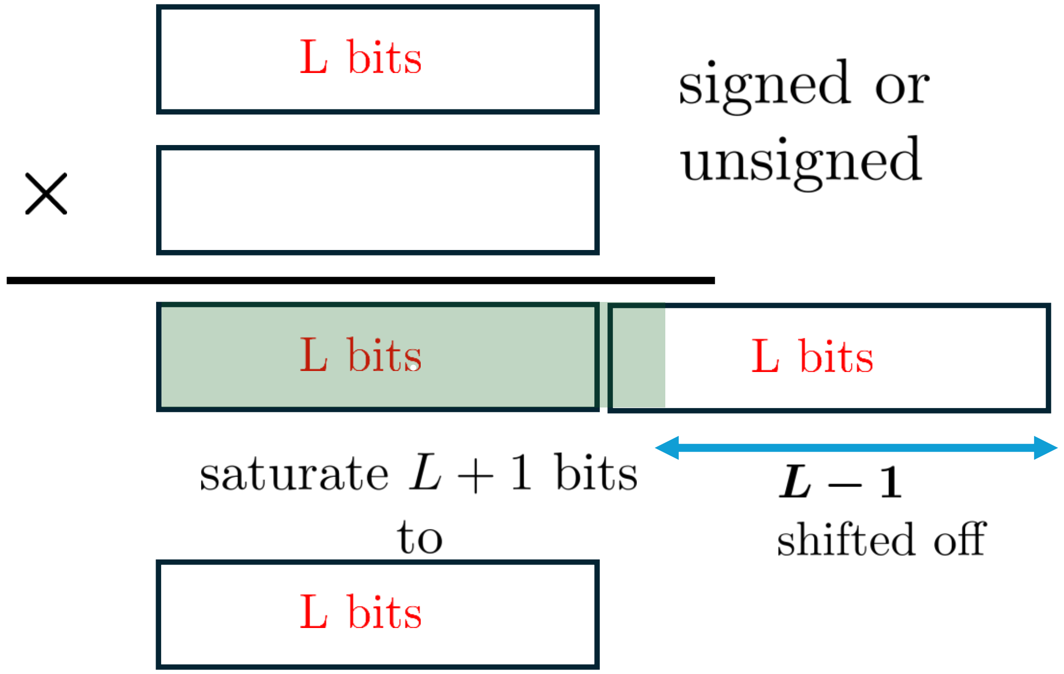

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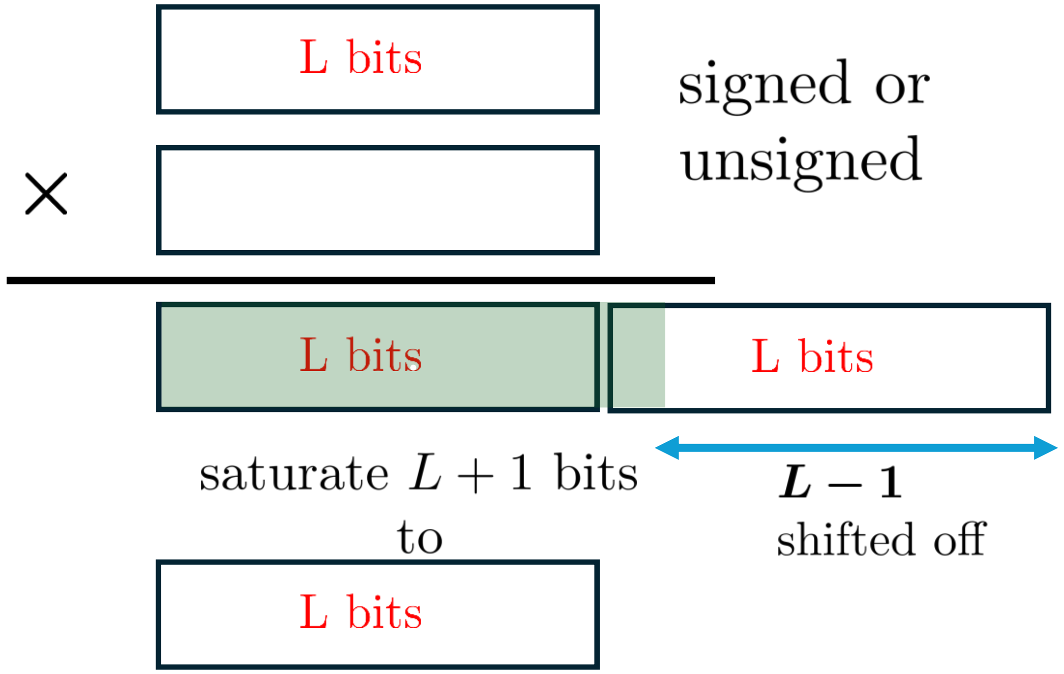
Fixed-point arithmetic can potentially carry 63 bits of precision



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Choice for scales are flexible: but 63 is nice

For example, Horner's recurrence:
$P_0 + R * (P_1 + R * (P_2 + \cdots))$

All P_j having the same scale and R being scaled at 63 eliminates the need for manual shifting

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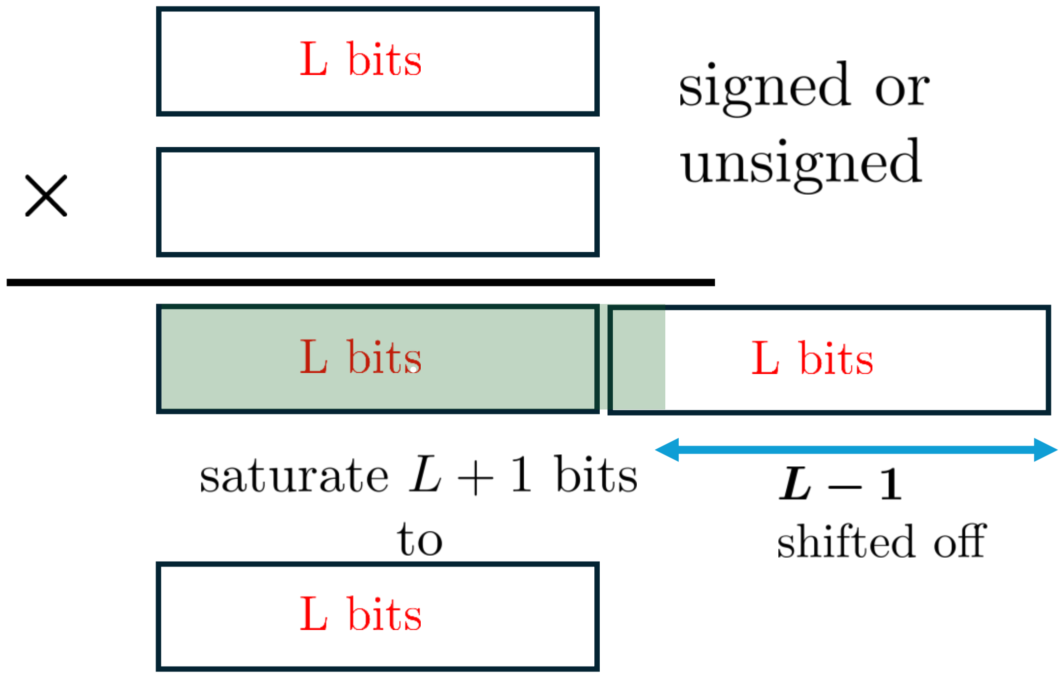
signed or unsigned



VecLibm: *Strategies and Illustrations*

- Exception Handling
- Precision Preservation

Exception Handling

- Challenge: Need to minimize branching while returning correct value AND signal.

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If there is some $x \in$ exceptional arguments

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If there is some $x \in$ exceptional arguments

set flag `x_special`
set result `y_special` for special inputs
substitute special inputs with safe values (e.g. 0.0 or 1.0)

Compute result `y_normal` for all (modified) inputs
`vmerge(y_normal, y_special, x_special)`

Exception Handling: example

`log(x)`

Value	Result	Signal
$+\infty$	$+\infty$	None
qNaN	qNaN	None
sNaN, -ve	qNaN	invalid
± 0	$-\infty$	divide by 0

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```
class = vfclass(vx)
x_special = and(class,0x3BF) > 0
if (vcpop(x_special) > 0){
// handle exceptions
  ..substitute -ve with sNaN
  ..substitute +0 with -0
  y_special = vfadd(vx, vfrec7(vx))
  ..substitute special x with 1.0
}
```

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}

...compute with vx as input
...yielding y_normal as result
y_result = vmerge(y_normal,
                  y_special, x_special)
```

Exception Handling: example

`asinpi(x)`

Value	Result	Signal
qNaN	qNaN	None
sNaN, $\lvert x\rvert > 1$	qNaN	invalid
± 1	$\pm\frac{1}{2}$	None

Exception Handling: example

`asinpi(x)`

Value	Result	Signal
qNaN	qNaN	None
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```
expo = ((vx >> 52) & 0x7FF)
x_special = expo >= 0x3FF
if (vcpop(x_special) > 0){
//handle exceptions
  ..substitute |x|>1 with sNaN
  ..substitute +-1 with +-1/4
  y_special = vfadd(vx, vx)
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Precision Preservation

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- General algorithmic approach well understood

Three steps to compute $\exp(x)$

$$
\begin{aligned}
r &\approx x - n\log(2); && \text{reduction} \\
p &\approx \exp(r); && \text{approximation} \\
e^x &\approx 2^n p && \text{reconstruction}
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Operations	# ops, an op is $+, -, \times$ or $\texttt{fmadd}$
$d(dd) + d(dd) \rightarrow dd$	6, 7, 8
$d(dd) \times d(dd) \rightarrow dd$	2, 3, 4
$d(dd)/d(dd) \rightarrow dd$	3, 4, 5 plus 1 div
$\sqrt{d(dd)} \rightarrow dd$	3, 4 plus 1 sqrt and 1 div

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Fast2Sum: (3 ops, not 6)
$S := \texttt{vfadd}(A, B)$
$s := \texttt{vfadd}(\texttt{vfsub}(A, S), B)$

Works if $|A| \geq |B|$
also works if $\mathrm{lsb}(A) \geq \mathrm{lsb}(B)$

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Fast2FMA: $AB + C$ (3 ops)
$S := \texttt{vfmadd}(A, B, C)$
$s := \texttt{vfmadd}(A, B, \texttt{vfsub}(C, S))$

Works if $C - S$ is exact

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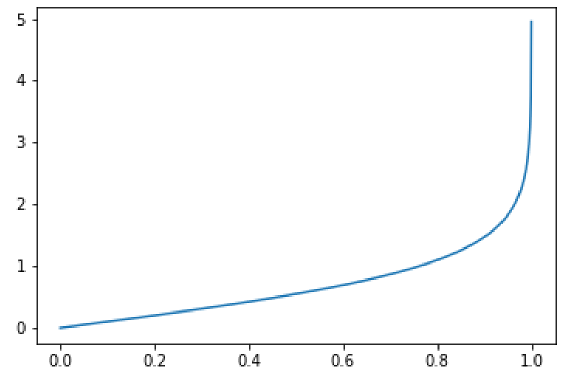
For example

$$
\begin{aligned}
y &:= p_k + r \times (p_{k+1} + r \times (p_{k+2} + \cdots)) \text{ in floating-point} \\
Y &:= \texttt{vfcvt\_x}(\texttt{vfmul}(2^q, y)) \text{ (convert to fixed point)} \\
Y &:= P_0 + R \times (P_1 + R \times (P_2 + ... + R \times (P_{k-1} + R \times Y))) \\
y &:= \texttt{vfmul}(\texttt{vfcvt\_f}(Y), 2^{-q})
\end{aligned}
$$

Precision Preservation: An example

Compute for $0 < x < 1$

$$\operatorname{atanh}(x) = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right) \qquad \log(y) = 2\operatorname{atanh}\left(\frac{y-1}{y+1}\right)$$



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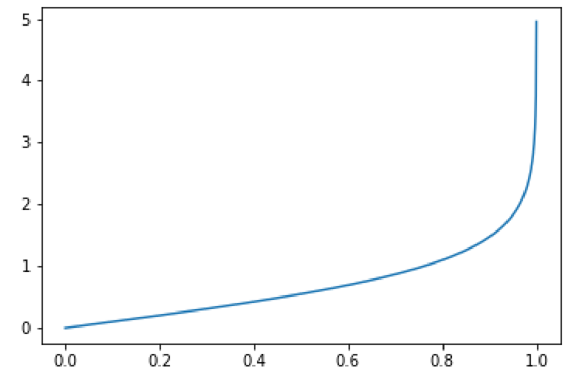

Some implementations:

get $1 \pm x$ and their quotient in dd

feed to special `log` that takes dd inputs

which in turn scales and transform in dd

computes poly. approx. of atanh

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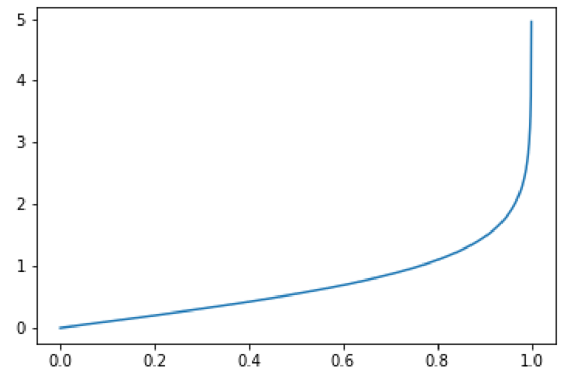
$$= (n/2)\log(2) + \frac{1}{2}\log\left(s\frac{1+x}{1-x}\right),\ s = 2^{-n}\ (n \text{ cheap to get})$$



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VecLibm implementation:

use `vfrec7` to get n

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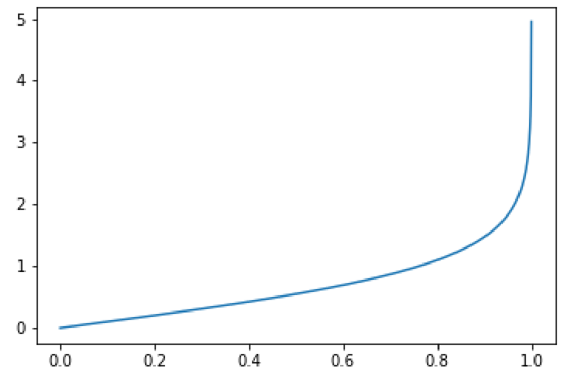
$$= (n/2)\log(2) + \operatorname{atanh}\left(\frac{(1+x)-(1-x)/s}{(1+x)+(1-x)/s}\right)$$



Some implementations:
- get $1 \pm x$ and their quotient in dd
- feed to special `log` that takes dd inputs
- which in turn scales and transform in dd
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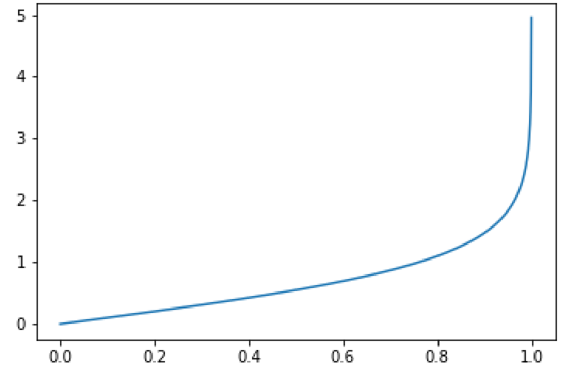
VecLibm implementation:
- use `vfrec7` to get n

Precision Preservation: An example



Compute for $0 < x < 1$

$$\operatorname{atanh}(x) = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right) \qquad \log(y) = 2\operatorname{atanh}\left(\frac{y-1}{y+1}\right)$$

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Fixed-point scale 60: $(1+X) \pm ((1-X) << n)$ yields exact value

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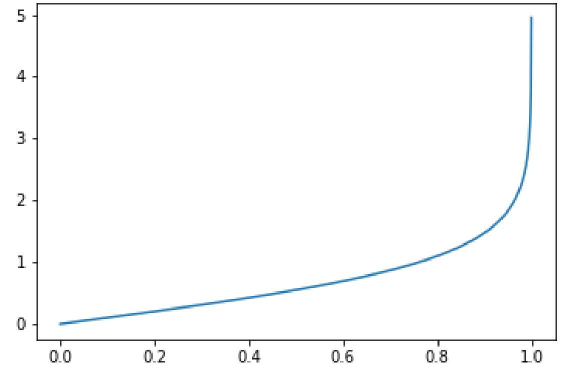

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Fixed-point NUMER, DENOM
$\rightarrow$ (a_hi,a_lo), (b_hi,b_lo)

Obtain dd quotient (r, r_lo)

$$(n/2)\log(2) + r + r\_lo + r^3 p(r^2)$$

VecLibm implementation:
- use `vfrec7` to get n
- computes numer, denom in fixed point
- get extra-precise FP input to atanh poly
- use floating-point computation onwards

VecLibm: Current Status At-a-Glance

Library Functions				Maximum Deviation in ulps			
exp	exp2	exp10	expm1	0.56	0.56	0.75	0.77
	when result underflows			0.77	0.77	0.82	N/A
log	log2	log10	log1p	0.55	0.57	0.56	0.66
pow	cbrt			0.55	0.52		
sin	sinpi	cos	cospi	0.79	0.76	0.76	0.77
tan	tanpi			0.62	0.61		
sinh	cosh	tanh		0.67	0.59	0.76	
asin	asinpi	acos	acospi	0.66	0.71	0.64	0.65
atan	atanpi	atan2	atan2pi	0.55	0.55	0.55	0.55
atan2pi underflows				0.75			
asinh	acosh	atanh		0.55	0.56	0.54	

https://github.com/rivosinc/veclibm