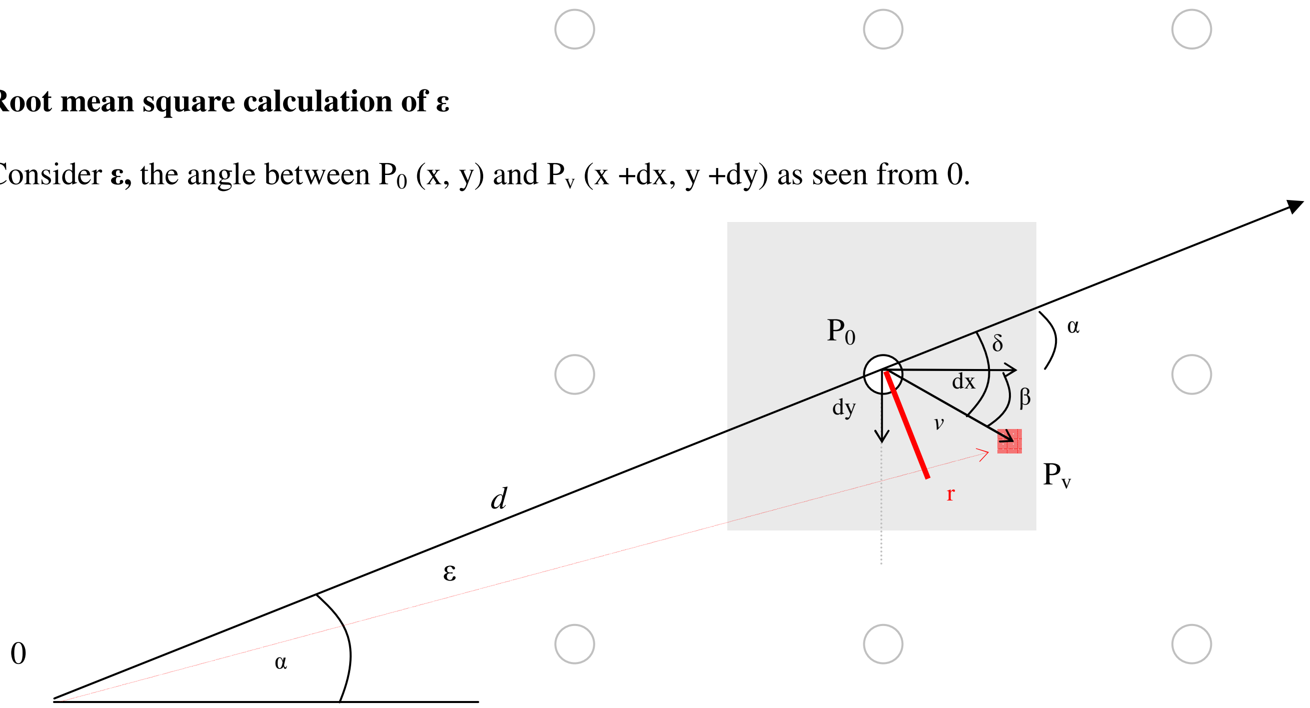
Root mean square calculation of ε

Consider **ε,** the angle between P_0 (x, y) and P_v (x +dx, y +dy) as seen from 0.



Note that δ = α-β, and *r*, the perpendicular projection of *v* to the observer (red line), is:

$$r = v \cdot \sin(\delta) = \sqrt{dx^2 + dy^2} \cdot \sin(\alpha - \beta)$$

If *d* is large and both *v* and *r* are small, then ε ≈ < tan(ε), and

$$\varepsilon \approx\le \frac{r}{d} = \frac{\sqrt{dx^2 + dy^2} \cdot \sin\left(\arctan\left(\frac{y}{x} \right) - \arctan\left(\frac{dy}{dx} \right) \right)}{\sqrt{x^2 + y^2}}$$

The root mean square of ε, in the shadowed area *S,* of area 1, using grid step unit, can be analytically evaluated.

$$rms = \sqrt{\frac{1}{S} \iint_S \varepsilon^2 dS}$$

The following Matlab script has been used to integrate the expression:

```
syms x y dx dy
r=sqrt(dx^2+dy^2)*sin(atan(y/x)-atan(dy/dx))
epsilon2=(r/sqrt(x^2+y^2))^2
rms=sqrt (int(int(epsilon2,dx,-0.5,0.5),dy,-0.5,0.5) )
pretty(simple(rms))
```

And finally, the result is:

$$rms = \frac{\sqrt{3}}{6 \cdot d}$$